



Robust H_∞ output feedback control for uncertain complex delayed dynamical networks

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ABSTRACT

In this paper, the robust H_∞ control problem of uncertain complex delayed dynamical networks with non-identical nodes is investigated. The dynamic output feedback controllers are designed to ensure robustly global stability and a smaller prescribed H_∞ disturbance attenuation level for the resulting closed-loop systems. The controller design problem can be solved by a cone complementary linearization algorithm involving linear matrix inequality (LMI) conditions. A numerical example is given to illustrate the effectiveness of the proposed method.

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1. Introduction

In recent years, complex networks have received a great deal of attention from researchers working in different fields. Many large-scale systems in nature and human societies can be described by complex networks with the nodes representing individuals in the systems and the edges representing the special connections among them, such as physical systems, the internet, communication networks, electricity distribution networks, genetic and biological networks, social networks, etc. [1–3]. In [4,5], the stability analysis and control of complex dynamical networks are investigated. In particular, special attention has been focused on the synchronization of complex networks [6–9]. The pinning control of the general complex dynamical networks was studied in [10,11]. Time delays often occur in many dynamical systems and may modify drastically the dynamic behavior of the systems. In [12–16], the synchronization stability criteria for general complex networks with time varying delays were studied. Furthermore, real-world complex networks often present uncertainties. The control problems of uncertain complex dynamical networks have been investigated in [17–20]. The impulsive control of the synchronization for uncertain complex dynamical networks was studied in [17]. Moreover, the stability of complex networks with different dynamical nodes by impulsive control was investigated in [18,21]. In [19,20], the adaptive control of uncertain complex dynamical networks was investigated. The existing results and methods on the control of uncertain complex dynamical networks are mainly based on the state feedback control technique, which needs the state of each node in the networks to be available. However, the state information may not be completely obtained in practice. This has motivated our research on the control of uncertain complex dynamical networks via dynamic output feedback.

Recently, the robust H_∞ control for various dynamical systems has been investigated in [22–25]. The existing results on the control of uncertain complex dynamical networks focus on the stabilization of the closed-loop systems with interior parametric uncertainties. The robust H_∞ control not only guarantees that the uncertain complex dynamical networks are robust for the parametric uncertainties, but also guarantees that the networks have a smaller prescribed H_∞ disturbance attenuation level with respect to external disturbances. However, little literature can be found to deal with the robust

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H_∞ control problem for the complex dynamical networks. In this paper, the robust H_∞ control problem for the uncertain complex delayed dynamical network with non-identical nodes is investigated. The dynamic output feedback controllers are designed such that the resulting closed-loop system is robustly globally stable and a prescribed H_∞ disturbance attenuation level is achieved. By a cone complementary linearization algorithm, an LMI-based approach is developed to tackle the addressed problem. Furthermore, an optimization algorithm is given to design the optimal H_∞ control which guarantees that the system has a smaller H_∞ disturbance attenuation. A numerical example shows the effectiveness of the proposed method.

The rest of this paper is organized as follows. In Section 2, an uncertain complex dynamical network with delayed coupling and some preliminaries are presented. In Section 3, the robust H_∞ output feedback control for the uncertain complex delayed dynamical network is considered. In Section 4, a numerical example shows the effectiveness of the proposed method. Conclusions are finally drawn in Section 5.

2. Problem formulation and preliminaries

Consider an uncertain complex dynamical network consisting of N non-identical nodes with diffusive and delayed coupling, in which each node of the network is an m -dimensional dynamical system. The state equations of the whole network are described by

$$\begin{cases} \dot{x}_i(t) = (A_i + \Delta A_i)x_i(t) + f_i(x_i(t)) + \sum_{j=1}^N c_{ij}\Gamma x_j(t - \tau(t)) + (B_i + \Delta B_i)u_i(t) + U_{1i}v_i(t) \\ y_i(t) = (D_{1i} + \Delta D_{1i})x_i(t) + (G_{1i} + \Delta G_{1i})u_i(t) + U_{2i}v_i(t) \\ z_i(t) = D_{2i}x_i(t) + G_{2i}u_i(t) \\ x_i(t) = \phi_i(t), \quad t \in [-h, 0], \quad i = 1, 2, \dots, N, \end{cases} \quad (1)$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{im}(t))^T \in R^m$ and $u_i(t) \in R^n$ are the state and control of the i -th node, respectively, $f: R^m \rightarrow R^m$ is a continuously differentiable function satisfying that

$$\|f_i(x) - f_i(y)\| \leq \mu_i \|x - y\|, \quad \forall x, y \in R^m, \quad (2)$$

where μ_i is a positive constant, $C = [c_{ij}]_{N \times N}$ represents the outer coupling configuration of the complex network, in which c_{ij} is defined as follows: if there is a connection from node j to node i ($i \neq j$), $c_{ij} > 0$, otherwise $c_{ij} = 0$, and the entries of matrix C satisfy the diffusive condition $c_{ii} = -\sum_{j=1, j \neq i}^N c_{ij}$, $\Gamma \in R^{m \times m}$ is the inner coupling matrix in each node, $v_i(t) \in R^p$ is the disturbance input which is square integrable on $[0, \infty)$, i.e. $v_i(\cdot) \in L_2[0, \infty)$, $y(t) \in R^l$ is the measured output, $z(t) \in R^q$ is the controlled output, the coupling delay $\tau(t)$ satisfies

$$\tau(t) \geq 0, \quad \dot{\tau}(t) \leq d < 1,$$

$\Delta A_i, \Delta B_i, \Delta D_{1i}, \Delta G_{1i}$ are unknown real norm-bounded matrix valued functions, representing time-varying parameter uncertainties of the form

$$\begin{bmatrix} \Delta A_i & \Delta B_i \\ \Delta D_{1i} & \Delta G_{1i} \end{bmatrix} = \begin{bmatrix} E_{1i} \\ E_{2i} \end{bmatrix} F(t) [N_{1i}, N_{2i}], \quad (3)$$

where $E_{1i}, E_{2i}, N_{1i}, N_{2i}$ are known real constant matrices, $F(t)$ is an unknown real time-varying matrix satisfying $F^T(t)F(t) \leq I$.

For the uncertain complex dynamical network (1), we consider a full-order dynamic output feedback controller with the following form

$$\begin{cases} \dot{\hat{x}}_i(t) = A_{ci}\hat{x}_i(t) + B_{ci}y_i(t) \\ u_i(t) = C_{ci}\hat{x}_i(t) \end{cases} \quad (4)$$

where $\hat{x}_i(t) \in R^m$ is the controller state. The matrices A_{ci}, B_{ci}, C_{ci} are to be determined.

Applying this controller to system (1), one can obtain the following closed-loop system

$$\begin{cases} \dot{\eta}_i(t) = (\bar{A}_i + \Delta \bar{A}_i)\eta_i(t) + Hf_i(x_i(t)) + \sum_{j=1}^N c_{ij}H\Gamma x_j(t - \tau(t)) + \bar{U}_i v_i(t) \\ z_i(t) = \bar{D}_i \eta_i(t), \end{cases} \quad (5)$$

where $\eta_i(t) = [x_i^T(t), \hat{x}_i^T(t)]^T$, $H = [I, 0]^T$, and

$$\begin{aligned} \bar{A}_i &= \begin{bmatrix} A_i & B_i C_{ci} \\ B_{ci} D_{1i} & A_{ci} + B_{ci} G_{1i} C_{ci} \end{bmatrix}, & \bar{U}_i &= \begin{bmatrix} U_{1i} \\ B_{ci} U_{2i} \end{bmatrix}, \\ \Delta \bar{A}_i &= \begin{bmatrix} \Delta A_i & \Delta B_i C_{ci} \\ B_{ci} \Delta D_{1i} & B_{ci} \Delta G_{1i} C_{ci} \end{bmatrix}, & \bar{D}_i &= \begin{bmatrix} D_{2i} & G_{2i} C_{ci} \end{bmatrix}. \end{aligned}$$

The robust H_∞ output feedback control for the uncertain complex network with coupling delay in this paper can be formulated as follows: for the closed-loop system (5), given a scalar $\gamma > 0$, find the dynamic output feedback controller (4) such that the following conditions are satisfied:

- (1) the closed-loop system (5) is robustly globally stable;
- (2) under the zero initial condition, the controlled output z_i satisfies

$$\sum_{i=1}^N \|z_i\|_2^2 \leq \gamma^2 \sum_{i=1}^N \|v_i\|_2^2, \quad (6)$$

for any nonzero $v_i(\cdot) \in L_2[0, \infty)$, $i = 1, 2, \dots, N$, and all admissible uncertainties.

In this case, the closed-loop system (5) is said to be globally stable with the disturbance attenuation γ .

To obtain the main results, the following lemma is introduced.

Lemma 2.1 ([26]). Given matrices G , M and L of appropriate dimension with G symmetric, then $G + MWL + L^T W^T M^T < 0$ for all matrices W satisfying $W^T W \leq I$, if and only if there exists some $\varepsilon > 0$ such that

$$G + \varepsilon^{-1} M M^T + \varepsilon L^T L < 0.$$

3. Robust H_∞ output feedback control

The following theorem provides a sufficient condition for the robust H_∞ output feedback control for the uncertain complex delayed dynamical network with non-identical nodes.

Theorem 3.1. The closed-loop system (5) is robustly globally stable with the disturbance attenuation γ , if there exist positive scalars ε_{1i} , ε_{2i} , ε_3 , positive definite matrices P_i , Q_i , such that the following matrix inequalities hold

$$\begin{bmatrix} P_i \bar{A}_i + \bar{A}_i^T P_i & 0 & P_i \bar{U}_i & H & P_i \bar{E}_i & \bar{N}_i & P_i H & \mu_i H & P_i H & \bar{D}_i^T & 0 \\ 0 & (d-1)Q_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma^T \\ \bar{U}_i^T P_i & 0 & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ H^T & 0 & 0 & -Q_i^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{E}_i^T P_i & 0 & 0 & 0 & -\varepsilon_{1i}^{-1} I & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{N}_i^T & 0 & 0 & 0 & 0 & -\varepsilon_{1i} I & 0 & 0 & 0 & 0 & 0 \\ H^T P_i & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2i}^{-1} I & 0 & 0 & 0 & 0 \\ \mu_i H^T & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2i} I & 0 & 0 & 0 \\ H^T P_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\varepsilon_3^{-1} I}{c_i} & 0 & 0 \\ \bar{D}_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ 0 & \Gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\varepsilon_3}{N} I \end{bmatrix} < 0 \quad (7)$$

where

$$c_i = \sum_{j=1}^N c_{ij}^2, \quad \bar{E}_i = \begin{bmatrix} E_{1i} \\ B_{ci} E_{2i} \end{bmatrix}, \quad \bar{N}_i = \begin{bmatrix} N_{1i}^T \\ C_{ci}^T N_{2i}^T \end{bmatrix}, \quad i = 1, 2, \dots, N.$$

Proof. To establish the stability condition for the closed-loop system (5), construct the following Lyapunov–Krasovskii function candidate for system (5) with $v(t) = 0$ as

$$V(t) = \sum_{i=1}^N \left[\eta_i^T(t) P_i \eta_i(t) + \int_{t-\tau(t)}^t \eta_i^T(s) H Q_i H^T \eta_i(s) ds \right].$$

Calculating the derivative of $V(t)$ along the trajectory of system (5) with $v(t) = 0$ yields

$$\begin{aligned} \frac{dV(t)}{dt} &= \sum_{i=1}^N [2\eta_i^T(t) P_i \dot{\eta}_i(t) + \eta_i^T(t) H Q_i H^T \eta_i(t) - (1 - \dot{\tau}(t)) x_i^T(t - \tau(t)) Q_i x_i(t - \tau(t))] \\ &\leq \sum_{i=1}^N 2\eta_i^T(t) P_i \left[(\bar{A}_i + \Delta \bar{A}_i) \eta_i(t) + H f_i(x_i(t)) + \sum_{j=1}^N c_{ij} H \Gamma x_j(t - \tau(t)) \right] \\ &\quad + \sum_{i=1}^N [\eta_i^T(t) H Q_i H^T \eta_i(t) - (1 - d) x_i^T(t - \tau(t)) Q_i x_i(t - \tau(t))]. \end{aligned} \quad (8)$$

By condition (2), together with the fact that $A^T B + B^T A \leq \alpha A^T A + \alpha^{-1} B^T B$, $\forall \alpha > 0$, and matrices A and B with appropriate dimensions, one can obtain

$$2\eta_i^T(t)P_i\Delta\bar{A}_i\eta_i(t) \leq \varepsilon_{1i}\eta_i^T(t)P_i^T\bar{E}_i\bar{E}_i^TP_i\eta_i(t) + \varepsilon_{1i}^{-1}\eta_i^T(t)\bar{N}_i\bar{N}_i^T\eta_i(t), \quad (9)$$

$$2\eta_i^T(t)P_iHf_i(x_i(t)) \leq \varepsilon_{2i}\eta_i^T(t)P_iHH^TP_i\eta_i(t) + \varepsilon_{2i}^{-1}\mu_i^2\eta_i^T(t)HH^T\eta_i(t), \quad (10)$$

$$2\sum_{i=1}^N\eta_i^T(t)P_i\sum_{j=1}^Nc_{ij}H\Gamma x_j(t-\tau(t)) \leq \sum_{i=1}^N\left[\eta_i^T(t)\varepsilon_3\left(\sum_{j=1}^Nc_{ij}^2\right)P_iHH^TP_i\eta_i(t) + x_i^T(t-\tau(t))N\varepsilon_3^{-1}\Gamma^T\Gamma x_i(t-\tau(t))\right]. \quad (11)$$

Combining (8)–(11), by Lemma 2.1, we can obtain that

$$\frac{dV(t)}{dt} \leq \sum_{i=1}^N\left[\eta_i(t) \atop x_i(t-\tau(t))\right]^T\begin{bmatrix}\Phi_{1i} & 0 \\ 0 & \Phi_{2i}\end{bmatrix}\begin{bmatrix}\eta_i(t) \\ x_i(t-\tau(t))\end{bmatrix}, \quad (12)$$

where

$$\begin{aligned}\Phi_{1i} &= P_i\bar{A}_i + \bar{A}_i^TP_i + \varepsilon_{1i}P_i^T\bar{E}_i\bar{E}_i^TP_i + \varepsilon_{1i}^{-1}\bar{N}_i\bar{N}_i^T + \varepsilon_{2i}P_iHH^TP_i \\ &\quad + \varepsilon_{2i}^{-1}\mu_i^2HH^T + \varepsilon_3\left(\sum_{j=1}^Nc_{ij}^2\right)P_iHH^TP_i + HQ_iH^T, \\ \Phi_{2i} &= N\varepsilon_3^{-1}\Gamma^T\Gamma - (1-d)Q_i.\end{aligned}$$

By the Schur complement formula [27], it follows from condition (7) that

$$\begin{bmatrix}\Phi_{1i} & 0 \\ 0 & \Phi_{2i}\end{bmatrix} < 0. \quad (13)$$

Therefore, the closed-loop system (5) with $v(t) = 0$ is robustly globally stable.

Next, we shall show that the closed-loop system (5) satisfies (6) for all nonzero $v_i(\cdot) \in L_2[0, \infty)$. By conditions (9)–(11), calculating the derivative of $V(t)$ along the trajectory of the closed-loop system (5), we have

$$\begin{aligned}\frac{dV(t)}{dt} &= \sum_{i=1}^N\left[2\eta_i^T(t)P_i\dot{\eta}_i(t) + \eta_i^T(t)HQ_iH^T\eta_i(t) - (1-\dot{\tau}(t))x_i^T(t-\tau(t))Q_ix_i(t-\tau(t))\right] \\ &\leq \sum_{i=1}^N2\eta_i^T(t)P_i\left[(\bar{A}_i + \Delta\bar{A}_i)\eta_i(t) + Hf_i(x_i(t)) + \sum_{j=1}^Nc_{ij}H\Gamma x_j(t-\tau(t)) + \bar{U}_iv_i(t)\right] \\ &\quad + \sum_{i=1}^N\left[\eta_i^T(t)HQ_iH^T\eta_i(t) - (1-d)x_i^T(t-\tau(t))Q_ix_i(t-\tau(t))\right] \\ &\leq \sum_{i=1}^N\begin{bmatrix}\eta_i(t) \\ x_i(t-\tau(t)) \\ v_i(t)\end{bmatrix}^T\begin{bmatrix}\Phi_{1i} & 0 & P_i\bar{U}_i \\ 0 & \Phi_{2i} & 0 \\ \bar{U}_i^TP_i & 0 & 0\end{bmatrix}\begin{bmatrix}\eta_i(t) \\ x_i(t-\tau(t)) \\ v_i(t)\end{bmatrix}. \quad (14)\end{aligned}$$

Then, by the Schur complement [27], condition (7) implies

$$\begin{bmatrix}\Phi_{1i} + \bar{D}_i^T\bar{D}_i & 0 & P_i\bar{U}_i \\ 0 & \Phi_{2i} & 0 \\ \bar{U}_i^TP_i & 0 & -\gamma^2I\end{bmatrix} < 0. \quad (15)$$

It follows from (14) and (15) that

$$\frac{dV(t)}{dt} + \sum_{i=1}^N(z_i^Tz_i - \gamma^2v_i^Tv_i) < 0. \quad (16)$$

Thus, under the zero initial condition, integrating both sides of (16) from 0 to $+\infty$, and noting that system (5) is stable, we have

$$\sum_{i=1}^N\|z_i\|_2^2 < \gamma^2\sum_{i=1}^N\|v_i\|_2^2. \quad (17)$$

The system (5) satisfies (6) for all nonzero $v_i(\cdot) \in L_2[0, \infty)$. This completes the proof. \square

It is noted that condition (7) is not an LMI, and cannot be solved by Matlab LMI Toolbox directly. In the following, a method of changing variables is applied to reduce condition (7) to LMIs for the given scalars $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_3$.

Partition P_i and its inverse as

$$P_i = \begin{bmatrix} Y_i & R_i \\ R_i^T & W_i \end{bmatrix}, \quad P_i^{-1} = \begin{bmatrix} X_i & M_i \\ M_i^T & Z_i \end{bmatrix}.$$

Let

$$F_{1i} = \begin{bmatrix} X_i & I \\ M_i^T & 0 \end{bmatrix}, \quad F_{2i} = \begin{bmatrix} I & Y_i \\ 0 & R_i^T \end{bmatrix}.$$

We have

$$P_i F_{1i} = F_{2i}.$$

Define the new controller variables as

$$\begin{aligned} \bar{A}_{ci} &= Y_i A_i X_i + R_i B_{ci} D_{1i} X_i + Y_i B_i C_{ci} M_i^T + R_i A_{ci} M_i^T + R_i B_{ci} G_{1i} C_{ci} M_i^T, \\ \bar{B}_{ci} &= R_i B_{ci}, \quad \bar{C}_{ci} = C_{ci} M_i^T. \end{aligned} \quad (18)$$

Therefore, the controller matrices A_{ci}, B_{ci}, C_{ci} can be uniquely determined by $\bar{A}_{ci}, \bar{B}_{ci}, \bar{C}_{ci}$.

In the following, we will state our main results on the robust H_∞ output feedback controller design based on LMIs approach.

Theorem 3.2. Consider the closed-loop system (5). Suppose that for some given positive scalars $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_3$, there exist positive definite matrices X_i, Y_i, T_i , and any matrices $\bar{A}_{ci}, \bar{B}_{ci}, \bar{C}_{ci}$ with appropriate dimensions such that the following LMIs hold

$$J_i = \begin{bmatrix} \Omega_i & 0 & J_{1i} & J_{2i} & J_{3i} & J_{4i} & J_{5i} & \mu_i J_{6i} & J_{7i} & J_{8i} & 0 \\ 0 & (d-1)T_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_i \Gamma^T \\ J_{1i}^T & 0 & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ J_{2i}^T & 0 & 0 & -T_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ J_{3i}^T & 0 & 0 & 0 & -\varepsilon_{1i}^{-1} I & 0 & 0 & 0 & 0 & 0 & 0 \\ J_{4i}^T & 0 & 0 & 0 & 0 & -\varepsilon_{1i} I & 0 & 0 & 0 & 0 & 0 \\ J_{5i}^T & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2i}^{-1} I & 0 & 0 & 0 & 0 \\ \mu_i J_{6i}^T & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2i} I & 0 & 0 & 0 \\ J_{7i}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\varepsilon_3^{-1}}{c_i} I & 0 & 0 \\ J_{8i}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ 0 & \Gamma T_i^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\varepsilon_3}{N} I \end{bmatrix} < 0, \quad (19)$$

$$\begin{bmatrix} X_i & I \\ I & Y_i \end{bmatrix} > 0, \quad i = 1, 2, \dots, N, \quad (20)$$

where

$$\begin{aligned} \Omega_i &= \begin{bmatrix} A_i X_i + X_i^T A_i^T + B_i \bar{C}_{ci} + \bar{C}_{ci}^T B_i^T & A_i + \bar{A}_{ci}^T \\ A_i^T + \bar{A}_{ci} & Y_i A_i + \bar{B}_{ci} D_{1i} + A_i^T Y_i^T + D_{1i}^T \bar{B}_{ci}^T \end{bmatrix}, \\ J_{1i} &= \begin{bmatrix} U_{1i} \\ Y_i U_{1i} + \bar{B}_{ci} U_{2i} \end{bmatrix}, \quad J_{2i} = J_{6i} = \begin{bmatrix} X_i \\ I \end{bmatrix}, \quad J_{3i} = \begin{bmatrix} E_{1i} \\ Y_i E_{1i} + \bar{B}_{ci} E_{2i} \end{bmatrix}, \\ J_{4i} &= \begin{bmatrix} X_i N_{1i}^T + \bar{C}_{ci}^T N_{2i}^T \\ N_{1i}^T \end{bmatrix}, \quad J_{5i} = J_{7i} = \begin{bmatrix} I \\ Y_i \end{bmatrix}, \quad J_{8i} = \begin{bmatrix} X_i D_{2i}^T + \bar{C}_{ci}^T G_{2i}^T \\ D_{2i}^T \end{bmatrix}. \end{aligned}$$

Then the complex network (1) with controller (4) is robustly globally stable with the disturbance attenuation γ . Moreover, the controller parameters are given by

$$\begin{aligned} A_{ci} &= R_i^{-1} (\bar{A}_{ci} - Y_i A_i X_i - R_i B_{ci} D_{1i} X_i - Y_i B_i C_{ci} M_i^T - R_i B_{ci} G_{1i} C_{ci} M_i^T) M_i^{-T}, \\ B_{ci} &= R_i^{-1} \bar{B}_{ci}, \quad C_{ci} = \bar{C}_{ci} M_i^{-T}, \end{aligned} \quad (21)$$

where R_i and M_i are any nonsingular matrices satisfying

$$M_i R_i^T = I - X_i Y_i. \quad (22)$$

Proof. Consider the change of controller variables (18). Let $T_i = Q_i^{-1}$. Pre and post-multiplying both sides of (7) by $\text{diag}\{F_{1i}^T, Q_i^{-1}, I, I, I, I, I, I, I, I, I\}$ and $\text{diag}\{F_{1i}, Q_i^{-1}, I, I, I, I, I, I, I, I, I\}$, respectively, then (19) can be obtained. Moreover, (20) implies that there exist nonsingular matrices R_i and M_i satisfying (22). Thus, the controller parameters in (21) can be obtained from (18). \square

Remark 3.1. Note that $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_3$ and their inverses occur in (19) and when the scalars $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_3$ are fixed, (19) is an LMI. For the given $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_3$, the feasibility of condition (19) can be solved by using Matlab LMI toolbox. However, this method may increase the conservativeness of the design of the controller. To reduce the conservativeness, $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_3$ should be regarded as variables. In fact, by using the cone complementary linearization algorithm, condition (19) can be expressed in terms of LMI.

Let $\theta_{1i} = \varepsilon_{1i}^{-1}, \theta_{2i} = \varepsilon_{2i}^{-1}, \theta_3 = \varepsilon_3^{-1}$. Condition (19) can be replaced by

$$\bar{J}_i < 0, \quad \varepsilon_{1i}\theta_{1i} = \varepsilon_{2i}\theta_{2i} = \varepsilon_3\theta_3 = 1, \quad i = 1, 2, \dots, N \quad (23)$$

where \bar{J}_i and J_i have the same structure with $\theta_{1i}, \theta_{2i}, \theta_3$ in place of $\varepsilon_{1i}^{-1}, \varepsilon_{2i}^{-1}, \varepsilon_3^{-1}$, respectively. Then conditions (19) and (20) in Theorem 3.2 can be considered as a cone complementary problem involving LMI conditions as follows.

$$\min \sum_{i=1}^N (\varepsilon_{1i}\theta_{1i} + \varepsilon_{2i}\theta_{2i}) + \varepsilon_3\theta_3 \quad (24)$$

subject to

$$\bar{J}_i < 0, \quad \begin{bmatrix} X_i & I \\ I & Y_i \end{bmatrix} > 0, \\ \begin{bmatrix} \varepsilon_{1i} & 1 \\ 1 & \theta_{1i} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \varepsilon_{2i} & 1 \\ 1 & \theta_{2i} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \varepsilon_3 & 1 \\ 1 & \theta_3 \end{bmatrix} \geq 0, \quad i = 1, 2, \dots, N.$$

To solve the above cone complementary problem, similar to the algorithm proposed in [28], we design the following algorithm.

Algorithm 3.1.

- Step 1. Given a scalar $\gamma > 0$, find a feasible solution $\varepsilon_{1i}(1), \varepsilon_{2i}(1), \varepsilon_3(1), \theta_{1i}(1), \theta_{2i}(1), \theta_3(1)$ satisfying the constraints in (24). Set $k = 1$ and go to Step 2.
- Step 2. Solve the cone complementary problem involving LMI conditions for the variables $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_3, \theta_{1i}, \theta_{2i}, \theta_3$:

$$W(k) = \min \sum_{i=1}^N (\varepsilon_{1i}(k)\theta_{1i} + \varepsilon_{1i}\theta_{1i}(k) + \varepsilon_{2i}(k)\theta_{2i} + \varepsilon_{2i}\theta_{2i}(k)) + \varepsilon_3(k)\theta_3 + \varepsilon_3\theta_3(k)$$

subject to the LMIs in (24).

If the LMIs in (24) are infeasible, then exit.

- Step 3. Set $W(0) = 0$. Given a sufficiently small $\theta > 0$, if the condition $|W(k) - W(k-1)| < \theta$ is satisfied, then exit. Otherwise, set $k = k + 1$, and $\varepsilon_{1i}(k+1) = \varepsilon_{1i}, \varepsilon_{2i}(k+1) = \varepsilon_{2i}, \varepsilon_3(k+1) = \varepsilon_3, \theta_{1i}(k+1) = \theta_{1i}, \theta_{2i}(k+1) = \theta_{2i}, \theta_3(k+1) = \theta_3$, and go back to Step 2.

In practice, we often hope that the system has a smaller H_∞ disturbance attenuation γ . Then the following optimal H_∞ control problem is introduced.

$$\min_{(\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_3, X_i, Y_i, T_i, \bar{A}_{ci}, \bar{B}_{ci}, \bar{C}_{ci})} \gamma^2 \quad (25)$$

subject to (19)–(20).

To solve the above optimal H_∞ control problem (25), we give the following algorithm which can result in a controller with a less conservative H_∞ disturbance attenuation.

Algorithm 3.2.

- Step 1. Choose a sufficiently large initial $\gamma > 0$ such that there exists a feasible solution to LMIs in (24).
- Step 2. Solve the cone complementary problem (24) by Algorithm 3.1.
- Step 3. If the condition (20) or (23) is infeasible, then exit and output $\gamma = \gamma + \Delta\gamma$ and the corresponding solutions. Otherwise, decrease the positive scalar γ with the step size $\Delta\gamma$, i.e., $\gamma = \gamma - \Delta\gamma$ and go back to Step 2.

In view of the physical limitations of actuators in practice, the values of decision variables $X_i, Y_i, T_i, \bar{A}_{ci}, \bar{B}_{ci}, \bar{C}_{ci}$, $i = 1, 2, \dots, N$ should not be too large. For this purpose, one can add the constraints of decision variables to the cone complementary problem (24) in Step 2 in Algorithm 3.2, e.g. $X_i < \alpha I$ with some given constant $\alpha > 0$.

Using Algorithm 3.2, a smaller prescribed H_∞ disturbance attenuation level and the corresponding controller can be obtained.

4. Numerical example

In this section, a numerical example is given to illustrate the effectiveness of the proposed method.

Consider the complex network (1) with three nodes, in which each node is a nonlinear system exhibiting a double-scroll chaotic attractor [29,30]. The parameter matrices in network (1) are given by

$$A_1 = A_2 = A_3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.5 & -0.5 & -0.5 \end{bmatrix}, \quad f_1(x_1(t)) = \begin{bmatrix} 0 \\ 0 \\ 5 \ln(x_{11}(t) + \sqrt{x_{11}^2(t) + 1}) \end{bmatrix},$$

$$f_2(x_2(t)) = \begin{bmatrix} 0 \\ 0 \\ 5 \frac{1 - \exp(-x_{21})}{1 + \exp(-x_{21})} \end{bmatrix}, \quad f_3(x_3(t)) = \begin{bmatrix} 0 \\ 0 \\ 5 \tanh(x_{31}) \end{bmatrix},$$

$$B_i = \text{diag}\{5, 5, 5\}, \quad D_{1i} = D_{2i} = \text{diag}\{1, 2, 1\},$$

$$G_{1i} = G_{2i} = \text{diag}\{0.1, 0.1, 0.1\}, \quad U_{1i} = U_{2i} = \text{diag}\{0.5, 0.5, 0.5\}, \quad i = 1, 2, 3.$$

Assume the diffusive outer coupling matrix is

$$C = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 2 \\ 1 & 2 & -3 \end{bmatrix},$$

and the inner coupling matrix is given by $\Gamma = \text{diag}\{1, 1, 1\}$, the coupling delay $\tau(t) = \frac{1+\cos(2t)}{4}$, and $d = 0.5$.

The uncertainties of the complex network (1) are given as follows

$$E_{1i} = E_{2i} = N_{1i} = N_{2i} = \text{diag}\{0.1, 0.1, 0.1\}, \quad F(t) = \text{diag}\{\cos(t), \cos(t), \cos(t)\},$$

and the upper bound α on all the variables is chosen as 190.

We now apply the optimal H_∞ control method to the above system with controller (4). A sufficiently large initial H_∞ disturbance attenuation γ is chosen as $\gamma = 30$ and θ is chosen to be 0.00001. Performing Algorithm 3.2 with Matlab, the positive scalar γ is decreasing with the step size $\Delta\gamma = 0.1$ at each time and the final smaller H_∞ disturbance attenuation is obtained as $\gamma = 17.3$, and the corresponding controller gains can be obtained as

$$A_{c1} = \begin{bmatrix} -116.0392 & -4.0679 & -1.2464 \\ -0.9444 & -52.4743 & 0.2294 \\ 0.3308 & -0.2396 & -47.5104 \end{bmatrix}, \quad B_{c1} = \begin{bmatrix} 1.0818 & -191.7791 & 2.1158 \\ 36.0650 & 4.5716 & 88.6634 \\ -88.6453 & -0.4111 & 36.0630 \end{bmatrix},$$

$$C_{c1} = \begin{bmatrix} -0.1237 & -9.5239 & 24.0790 \\ 11.2511 & -0.6768 & 0.0801 \\ -0.2143 & -23.4143 & -9.7964 \end{bmatrix}, \quad A_{c2} = \begin{bmatrix} -121.3562 & -5.2142 & -1.4279 \\ -1.3263 & -44.636 & 0.2351 \\ 0.2794 & -0.2417 & -38.0526 \end{bmatrix},$$

$$B_{c2} = \begin{bmatrix} 1.0030 & -189.2043 & 2.1516 \\ 35.7096 & 4.5725 & 87.3280 \\ -87.3094 & -0.2339 & 35.7052 \end{bmatrix}, \quad C_{c2} = \begin{bmatrix} -0.1332 & -11.5991 & 29.2383 \\ 12.8275 & -0.8207 & 0.0652 \\ -0.2545 & -28.3659 & -11.9576 \end{bmatrix},$$

$$A_{c3} = \begin{bmatrix} -119.7280 & -4.7713 & -1.3549 \\ -1.1863 & -48.8583 & 0.2340 \\ 0.3012 & -0.2398 & -42.9450 \end{bmatrix}, \quad B_{c3} = \begin{bmatrix} 1.0303 & -188.7100 & 2.1299 \\ 35.5663 & 4.5549 & 87.1554 \\ -87.1369 & -0.3023 & 35.5627 \end{bmatrix},$$

$$C_{c3} = \begin{bmatrix} -0.1309 & -10.8286 & 27.3287 \\ 12.2799 & -0.7702 & 0.0732 \\ -0.2406 & -26.5360 & -11.1541 \end{bmatrix}.$$

Denote $x(t) = (x_1^T(t), x_2^T(t), x_3^T(t))^T$, $\hat{x}(t) = (\hat{x}_1^T(t), \hat{x}_2^T(t), \hat{x}_3^T(t))^T$. Let initial conditions $x(t) = (1, -1, 1, -2, 2, -1, -1, 1, 2)^T$, $\hat{x}(t) = (1, -1, 1, -1, 1, -1, -1, -1, 1)^T$, $t \leq 0$. The simulation results of the state responses for the complex network (1) with $v(t) = 0$ and the maximum allowable time-delay $\tau = 0.5$ via the dynamic output feedback controller (4) are given in Fig. 1.

Under the output feedback H_∞ control, the uncertain network (1) is robustly globally stable and a smaller disturbance attenuation γ is satisfied. Moreover, under zero initial condition, consider a disturbance input

$$v_i(t) = (e^{-t} \sin 20\pi t, e^{-t} \sin 20\pi t, e^{-t} \sin 20\pi t) \in L_2[0, \infty), \quad i = 1, 2, 3.$$

We can calculate that $\sum_{i=1}^3 \|z_i\|_2^2 = 1.63$ and $\sum_{i=1}^3 \|v_i\|_2^2 = 4.49$, and condition (6) is satisfied.

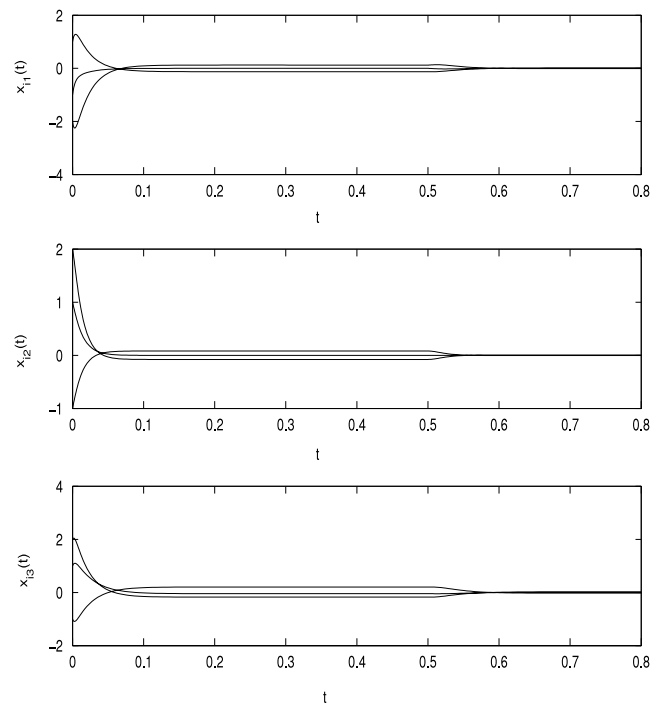


Fig. 1. The state responses $x_i(t)$ ($i = 1, 2, 3$) of the complex network (1) with $\tau = 0.5$ via the dynamic output feedback controller (4).

5. Conclusion

In this paper, the robust H_∞ output feedback control for the uncertain complex network with coupling delay is investigated. The H_∞ observer-based controllers have been designed to achieve a smaller H_∞ disturbance attenuation. The controller design problem can be solved by a cone complementary linearization algorithm involving LMI conditions. A numerical example shows the effectiveness of the proposed method.

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